# **Electrical behavior**

**Topic 3** 

# **Reading assignment**

- Chung, Multifunctional cementbased Materials, Ch. 2.
- Askeland and Phule, The Science and Engineering of Materials, 4<sup>th</sup> Ed., Chapter 18.

## Supplementary reading

Shackelford, Materials Science for Engineers, 6<sup>th</sup> Ed., Ch. 15.



Conducting range	Material	Conductivity, $\sigma$ ( $\Omega^{-1} \cdot m^{-1}$ )
Conductors	Aluminum (annealed)	$35.36 imes10^6$
	Copper (annealed standard)	$58.00 imes10^6$
	Iron (99.99 + %)	$10.30 imes10^6$
	Steel (wire)	$5.719.35\times10^6$
Semiconductors	Germanium (high purity)	2.0
	Silicon (high purity)	$0.40  imes 10^{-3}$
	Lead sulfide (high purity)	38.4
Insulators	Aluminum oxide	$10^{-10} - 10^{-12}$
	Borosilicate glass	10-13
	Polvethvlene	$10^{-13} - 10^{-15}$
	Nylon 66	$10^{-12} - 10^{-13}$
Source: Data from C.	A. Harper, Ed., Handbook of Materials	and Processes for
Electronics, M	cGraw-Hill Book Company, NY, 1970; a	and J. K. Stanley,
Electrical and	Magnetic Properties of Metals, Amer	ican Society for
Devolucion	to Materials Science for Engineers, Sinth Edition by Jum	es F. Shackelford,

Material	Conductivity (ohm <sup>-1</sup> - cm <sup>-1</sup> )
Superconductors	
Hg, Nb <sub>5</sub> Sn,	
YBa <sub>2</sub> Cu <sub>3</sub> O <sub>7-x</sub>	Infinite (under certain conditions
MgB <sub>2</sub>	such as low temperatures)
Metals	
Alkali metals	
Na	$2.13 \times 10^{5}$
ĸ	$1.64 \times 10^{6}$
Alkali earth metals:	
Mit	$2.25 \times 10^{5}$
Ca	$3.16 \times 10^{6}$
Group 3B metals	
A	$3.77 \times 10^{5}$
Ga	$0.66 \times 10^{5}$
Transition metals	
Fo	$1.00 \times 10^{6}$
Ni	$1.46 \times 10^{6}$
Group 1B metals	
Ou	$5.98 \times 10^{5}$
Ac	$6.80 \times 10^{5}$
Au	$4.26 \times 10^{6}$

Material	Conductivity (ohm <sup>-1</sup> · cm <sup>-1</sup> )
Semiconductors	
Group 4B elements:	
54	$5 \times 10^{-6}$
Ge	0.02
x-Sn	$0.9 \times 10^{6}$
Compound semiconductors	
GaAs	$2.5 \times 10^{-9}$
AIAs	0.1
SIC	10-10
Ionic Conductors	
Indium tin oxide (170)	
Yttria-stabilized zirconia (YSZ)	
Insulators, Linear and Nonlinear Diel	ectrics
Polymers	
Polyethylene	20-25
Polytetrafluorethylene	10-18
Polystyrene	10-17 to 10-19
EDOXY	10-12 to 10-17
Ceramics	575k AM253
Alumina (AluOa)	10-34
Silicate glasses	10-17
Boron nitride (BN)	10-13
Barium titanate (BaTiO <sub>3</sub> )	10-34
C (diamond)	<10 <sup>-18</sup>
* Unious specified otherwise, assumes	high pullty material.



Figure 18.2 (a) Charge carriers, such as electrons, are deflected by atoms or defects and take an irregular path through a conductor. The average rate at which the carriers move is the drift velocity v. (b) Valence electrons in the metallic bond move easily. (c) Covalent bonds must be broken in semiconductors and insulators for an electron to be able to move. (d) Entire ions must diffuse to carry charge in many ionically bonded materials.

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Mean free path – The average distance that electrons can move without being scattered by other atoms.











#### TABLE 18-2 Some useful relationships, constants, and units

Electron volt = 1 eV =  $1.6 \times 10^{-19}$  Joule =  $1.6 \times 10^{-12}$  erg 1 amp = 1 coulomb/second 1 volt = 1 amp - ohm k<sub>B</sub> T at room temperature (300 K) = 0.0259 eV c = speed of light 2.998 ×  $10^{-8}$  m/s  $\epsilon_0$  = perimitivity of free space =  $8.85 \times 10^{-12}$  F/m q = charge on electron =  $1.6 \times 10^{-19}$  C Avogadro's number N<sub>A</sub> =  $6.023 \times 10^{23}$ k<sub>B</sub> = Boltzmann's constant =  $8.63 \times 10^{-5}$  eV/K =  $1.38 \times 10^{-23}$  J/K h = Planck's constant  $6.63 \times 10^{-34}$  J-s =  $4.14 \times 10^{-15}$  eV-s Current density – The current flowing through per unit crosssectional area.



$$R = \frac{V}{I}$$
$$S = \frac{\ell}{A} \frac{I}{V} = \frac{(I/A)}{(V/\ell)}$$
$$S = \frac{\tilde{J}}{(V/\ell)}$$





$$\frac{dV}{dx} = -\frac{V}{\ell}$$
Electric field
$$E = -\frac{dV}{dx}$$

$$\frac{V}{\ell} = -\frac{dV}{dx} = E$$

$$s = \frac{\tilde{J}}{E}$$

Drift velocity  $v \propto E$   $v = \mu E$  where  $\mu$  is the mobility

- Drift velocity The average rate at which electrons or other charge carriers move through a material under the influence of an electric or magnetic field.
- Mobility The ease with which a charge carrier moves through a material.

- Current density The current flowing through per unit cross-sectional area.
- Electric field The voltage gradient or volts per unit length.
- Drift velocity The average rate at which electrons or other charge carriers move through a material under the influence of an electric or magnetic field.
- Mobility The ease with which a charge carrier moves through a material.
- Dielectric constant The ratio of the permittivity of a material to the permittivity of a vacuum, thus describing the relative ability of a material to polarize and store a charge; the same as relative permittivity.

$$I = qnvA$$
$$\widetilde{J} = \frac{I}{A} = \frac{qnvA}{A} = qnv$$
$$\boldsymbol{s} = \frac{\widetilde{J}}{E} = \frac{qnv}{E}$$

$$\frac{\mathbf{v}}{\mathbf{E}} = \mathbf{m}$$
$$\mathbf{\sigma} = \mathbf{q} \mathbf{n} \mathbf{\mu}$$

Flux  

$$J_{n} = -D_{n} \frac{dn}{dx}$$
Current density  

$$\widetilde{J}_{n} = (-q) J_{n}$$

$$\widetilde{J}_{n} = (-q) \left( -D_{n} \frac{dn}{dx} \right)$$

$$= q D_{n} \frac{dn}{dx} .$$

Flux  

$$J_{p} = -D_{p} \frac{dp}{dx}$$
Current density  

$$\tilde{J}_{p} = q J_{p} = q \left(-D_{p} \frac{dp}{dx}\right) = -q D_{p} \frac{dp}{dx} .$$





















- Valence band The energy levels filled by electrons in their lowest energy states.
- Conduction band The unfilled energy levels into which electrons can be excited to provide conductivity.
- Energy gap (Bandgap) The energy between the top of the valence band and the bottom of the conduction band that a charge carrier must obtain before it can transfer a charge.















- •Holes are in the valence band.
- Conduction electrons are in the conduction band.

Holes - Unfilled energy levels in the valence band. Because electrons move to fill these holes, the holes move and produce a current.





Radiative recombination -Recombination of holes and electrons that leads to emission of light; this occurs in direct bandgap materials.



Resistance due to component i  

$$R_{i} = r_{i} \frac{\ell}{A_{i}}$$
Current through component i  

$$I_{i} = \frac{VA_{i}}{r_{i}\ell}$$
Total current through the composite  

$$I = I_{1} + I_{2} + I_{3} = \frac{V}{\ell} \left( \frac{A_{1}}{r_{1}} + \frac{A_{2}}{r_{2}} + \frac{A_{3}}{r_{3}} \right)$$

Total resistance  

$$R = r_{||} \frac{\ell}{\left(A_{1} + A_{2} + A_{3}\right)}$$
Total current  

$$I = \frac{V}{R} = \frac{V(A_{1} + A_{2} + A_{3})}{r_{||}\ell}$$

$$\frac{V}{\ell} \left(\frac{A_{1}}{r_{1}} + \frac{A_{2}}{r_{2}} + \frac{A_{3}}{r_{3}}\right) = \frac{V}{r_{||}\ell} (A_{1} + A_{2} + A_{3})$$

$$\frac{1}{r_{||}} = \frac{1}{r_{1}} \frac{A_{1}}{(A_{1} + A_{2} + A_{3})} + \frac{1}{r_{2}} \frac{A_{2}}{(A_{1} + A_{2} + A_{3})} + \frac{1}{r_{3}} \frac{A_{3}}{(A_{1} + A_{2} + A_{3})} = \frac{1}{r_{1}} \frac{A_{3}}{f_{1} + A_{2} + A_{3}}$$
$$= \frac{1}{r_{1}} f_{1} + \frac{1}{r_{2}} f_{2} + \frac{1}{r_{3}} f_{3},$$
Rule of Mixtures



$$V_{i} = IR_{i}$$
$$= Ir_{i} \frac{L_{i}}{A}$$
Total voltage drop

$$\mathbf{V} = \frac{\mathbf{I}}{\mathbf{A}} (\boldsymbol{r}_1 \mathbf{L}_1 + \boldsymbol{r}_2 \mathbf{L}_2 + \boldsymbol{r}_3 \mathbf{L}_3)$$

Total resistance  

$$R = r_{\perp} \frac{\left(L_{1} + L_{2} + L_{3}\right)}{A}$$
Total voltage drop  

$$V = IR$$

$$= Ir_{\perp} \frac{\left(L_{1} + L_{2} + L_{3}\right)}{A}$$

Total voltage drop  

$$V = \frac{I}{A} (r_1 L_1 + r_2 L_2 + r_3 L_3)$$

$$V = IR$$

$$= Ir_{\perp} \frac{\left(L_1 + L_2 + L_3\right)}{A}$$

$$\frac{\mathrm{I}}{\mathrm{A}} (\mathbf{r}_{1} \mathrm{L}_{1} + \mathbf{r}_{2} \mathrm{L}_{2} + \mathbf{r}_{3} \mathrm{L}_{3})$$
$$= \mathrm{I} \mathbf{r}_{\perp} \frac{\left(\mathrm{L}_{1} + \mathrm{L}_{2} + \mathrm{L}_{3}\right)}{\mathrm{A}}$$







## **Percolation threshold**

Minimum volume fraction of conductive fibers (or particles) for adjacent fibers (or particles) to touch each other and form a continuous conductive path.











For an intrinsic semiconductor (n = p),

$$s = qn(m_h + m_p)$$

Current density due to both an electric field and a concentration gradient  $\widetilde{J}_n = qn \ \boldsymbol{m}_n \ \boldsymbol{E} + qD_n \frac{dn}{dx} \quad .$  $\widetilde{J}_p = qp \ \boldsymbol{m}_p \ \boldsymbol{E} - qD_p \frac{dp}{dx} \quad .$  $\widetilde{J} = \widetilde{J}_n + \widetilde{J}_p \quad .$ 

- Intrinsic semiconductor A semiconductor in which properties are controlled by the element or compound that makes the semiconductor and not by dopants or impurities.
- Extrinsic semiconductor A semiconductor prepared by adding dopants, which determine the number and type of charge carriers.
- Doping Deliberate addition of controlled amounts of other elements to increase the number of charge carriers in a semiconductor.













Dopant	Silicon		Germanium	
	Ed	E,	Ed	E,
Р	0.045		0.0120	
As	0.049		0.0127	
Sb	0.039		0.0096	
В		0.045		0.0104
Al		0.057		0.0102
Ga		0.065		0.0108
In		0.160		0.0112











 $\boldsymbol{n} = \frac{c}{l}$   $E \propto \boldsymbol{n}$   $E = h \boldsymbol{n} \quad \text{(Photon energy)}$ where h = Planck's constant  $= 6.6262 \text{ x } 10^{-34} \text{ J.s}$ 

Photon energy must be at least equal to the energy bandgap in order for electrons to be excited from the valence band to the conduction band. Consider an n-type semiconductor being illuminated.

Illumination increases conduction electrons and holes by equal number, since electrons and holes are generated in pairs.

Thus, the minority carrier concentration  $(p_n)$  is affected much more than the majority carrier concentration  $(n_n)$ .

Material	Energy gap, <i>Eg</i> (eV)	Electron mobility, $\mu_e[\mathbf{m}^2/(\mathbf{V}\cdot\mathbf{s})]$	Hole mobility, $\mu_h[\mathbf{m}^2/(\mathbf{V}\cdot\mathbf{s})]$	Carrier density $n_e (= n_h)$ $(m^{-3})$
Si	1.107	0.140	0.038	$14 imes 10^{15}$
Ge	0.66	0.364	0.190	$23 imes 10^{18}$
CdS	2.59ª	0.034	0.0018	
GaAs	1.47	0.720	0.020	$1.4 imes10^{12}$
InSb	0.17	8.00	0.045	$13.5 imes10^{21}$
aThi lim	nics, McGraw-H is value is above it is somewhat ar els that substant	ill Book Company, NY our upper limit of 2 eV bitrary. In addition, mo ially change the nature	(, 1970. used to define a semio st commercial devices of the band gap (see	conductor. Such a s involve impurity Chapter 17).
		Table 15.5		
Р	Properties of Some	Common Semiconducto	rs at Room Temperatur	e (300 K).

Seniconductor	Bandgap eV	$\frac{\text{Mobility of}}{\text{Electrons}} \left( \mu_{p} \right) \\ \frac{\text{cm}^{2}}{\text{V-s}}$	$\begin{array}{c} \text{Mobility of} \\ \text{Holes}\left(\mu_{p}\right) \\ \frac{\text{cm}^{2}}{\text{V-s}} \end{array}$	Dielectric Constant (k)	Resistivity Ω-cm	Density gm cm <sup>3</sup>	Nelting Temperature C
Silican (Si)	111	1350	480	11.8	25×30 <sup>4</sup>	233	1415
Amorphous Silicon (a:Si-H)	1,70	I	10-2	~118	1010	-2.30	-
Gemanium (Ge)	8.67	3900	1900	15.0	43	5.32	996
SC (a)	2.86	500		30.2	20 <sup>30</sup>	3.21	2830
Gallum Arsenide (GaAs)	1.43	8500	400	13.2	$4 \times 10^8$	5.31	1238
Diamond	~5.50	1800	1500	57	>10 <sup>18</sup>	3.52	~4200
e St	0.10	2000	1000	-	10-4	5.80	232



- Temperature Effect When the temperature of a metal increases, thermal energy causes the atoms to vibrate
- Effect of Atomic Level Defects -Imperfections in crystal structures scatter electrons, reducing the mobility and conductivity of the metal





Change of resistivity with temperature for a metal

$$\frac{\Delta r}{r} = a\Delta T$$

where  $\alpha$  = temperature coefficient of electrical resistivity



Metal	Room Temperature Resistivity (ohm · cm)	Temperature Resistivity Coefficient (ag) (ohm/ohm · °C)
Bo	$4.0 \times 10^{-6}$	0.0250
Mg	$4.45 \times 10^{-6}$	0.0037
Ca	$3.91 \times 10^{-6}$	0.0042
AL	$2.65 \times 10^{-6}$	0.0043
Cr	12.90 × 10 <sup>-6</sup> (0°C)	0.0030
Fe	$9.71 \times 10^{-6}$	0.0065
Co	$6.24 \times 10^{-6}$	0.0053
Ni	$6.84 \times 10^{-6}$	0.0069
Cu	$1.67 \times 10^{-6}$	0.0043
Ag	$1.59 \times 10^{-6}$	0.0041
Au	$2.35 \times 10^{-6}$	0.0035
Pd	$10.8 \times 10^{-6}$	0.0037
W	5.3 × 10 <sup>-6</sup> (27°C)	0.0045
Pt	$9.85 \times 10^{-6}$	0.0039

Material	Resistivity at 20° C $\rho_{rt}$ ( $\Omega \cdot m$ )	Temperature coefficien of resistivity at $20^{\circ}$ C $\alpha$ (° C <sup>-1</sup> )
Aluminum (annealed)	$28.28 \times 10^{-9}$	0.0039
Copper (annealed standard)	$17.24 \times 10^{-9}$	0.00393
Gold	$24.4 \times 10^{-9}$	0.0034
Iron (99.99+%)	$97.1 \times 10^{-9}$	0.00651
Lead (99.73+%)	$206.48 \times 10^{-9}$	0.00336
Magnesium (99.80%)	$44.6 \times 10^{-9}$	0.01784
Mercury	$958 \times 10^{-9}$	0.00089
Nickel (99.95% + Co)	$68.4 \times 10^{-9}$	0.0069
Nichrome (66% Ni + Cr and Fe)	$1,000 \times 10^{-9}$	0.0004
Platinum (99.99%)	$106 \times 10^{-9}$	0.003923
Silver (99.78%)	$15.9 \times 10^{-9}$	0.0041
Steel (wire)	$107 - 175 \times 10^{-9}$	0.006-0.0036
Tungsten	$55.1 \times 10^{-9}$	0.0045
Zinc	$59.16 \times 10^{-9}$	0.00419
Source: Data from J. K. Stanley, <i>Electr</i>	ical and Magnetic Properties	of Metals, American Society

Resistivities and Temperature Coefficients of Resistivity for Some Metallic Conductors







Matthiessen's rule – The resistivity of a metallic material is given by the addition of a base resistivity that accounts for the effect of temperature, and a temperature independent term that reflects the effect of atomic level defects, including impurities forming solid solutions.







Alley	$\frac{\sigma_{aboy}}{\sigma_{Co}} \times 100$	Remarks
Pure annieu/ed copper	100	Few detects to scatter electrons, the mean free path is long.
Pure cooper deformed 80%	98	Many dislocations, but because of the tangled nature of the dislocation networks, the mean free path is still long.
Dispersion strengthened Cu-0.7% AlyOy	85	The dispersed phase is not as closely spaced as solid-solution atoms, nor is it coherent, as in age hardening. Thus, the effect on conductivity is small.
Solution treated Cu 2% Be	18	The alkiy is single phase, however, the small amount of solid-solution strengthening from the supersaturated benylium greatly decreases conductively.
Aged Cu-2% Be	23	During aging, the benytium leaves the cooper lattice to produce a coherent precipitale. The precipitale does not interfere with conductivity as much as the solid-solution atoms.
Cu-35% Zn	28	This alley is solid-solution strengthened by zinc, which has an abors: radius near that of cooper. The conductivity is low, but not at low as when beryflum is present.



#### $\mathbf{s} = \mathbf{q} \mathbf{n} \mathbf{m}$

For a metal, s decreases with increasing temperature because  $\mu$  decreases with increasing temperature.

For a semiconductor, s increases with increasing temperature because n and/or p increases with increasing temperature.

#### For a semiconductor

 $n \propto e^{-E_g/2kT}$ ,

where  $E_g$  = energy band gap between conduction and valence bands, k = Boltzmann's constant, and T = temperature in K.

The factor of 2 in the exponent is because the excitation of an electron  $across E_g$  produces an intrinsic conduction electron and an intrinsic hole.

Taking natural logarithms,  $s = s_{o}e^{-E_{g}/2kT}$ .  $\ln s = \ln s_{o} - \frac{E_{g}}{2kT}$ . Changing the natural logarithms to logarithms of base 10,  $\log s = \log s_{o} - \frac{E_{g}}{(2.3)2kT}$ .

### Thermistor –

A semiconductor device that is particularly sensitive to changes in temperature, permitting it to serve as an accurate measure of temperature. Conductivity of an ionic solid

 $\boldsymbol{s} = q n \boldsymbol{m}_C + q n \boldsymbol{m}_A = q n (\boldsymbol{m}_C + \boldsymbol{m}_A)$ ,

where n = number of Schottky defects per unit volume  $\mu_C =$  mobility of cations,  $\mu_A =$  mobility of anions.

## An n-type semiconductor $n=n_i + n_e$ , where n = total concentration of conduction electrons, $n_i$ = concentration of intrinsic conduction electrons, $n_e$ = concentration of extrinsic conduction electrons.

$$\begin{split} D &\rightarrow D^{+} + e^{-} , \\ n_{e} &= N_{D} + , \\ n_{i} &\propto e^{-Eg/2 \, kT} & . \\ n_{e} &\propto e^{-E_{D}/kT} & . \\ n_{i} &< < n_{e} & . \\ p &= p_{i} & . \end{split}$$



No extrinsic holes, thus 
$$p = p_i \ .$$
 However, 
$$p_i = n_i$$
 Thus, 
$$p = n_i$$

 $n \cong n_{e}$   $p \cong 0 \quad .$   $\boldsymbol{s} = qn \ \boldsymbol{m}_{n} + qp \ \boldsymbol{m}_{p} \quad .$   $\boldsymbol{s} \cong qn \ \boldsymbol{m}_{n}$ 











$$A + e^{-} \rightarrow A^{-} ,$$
  

$$A \rightarrow A^{-} + h^{+} ,$$
  

$$p_{e} = N_{A^{-}} ,$$
  

$$p_{i} \propto e^{-Eg/2kT} ,$$
  

$$p_{e} \propto e^{-EA/kT} .$$



$$\begin{split} n &= n_i \ . \\ n &= p_i \ . \\ p &\cong p_e \\ n &\cong 0 \ . \end{split}$$
 before acceptor saturation

tion Production

# The mass-action law

Product of n and p is a constant for a particular semiconductor at a particular temperature Intrinsic semiconductor  $n = n_i = p_i = p \quad .$   $np = n_i^2 \quad .$   $n_i = 1.5 \times 10^{10} \text{ cm}^{-3} \text{ for Si}$   $n_i = 2.5 \times 10^{13} \text{ cm}^{-3} \text{for Ge} \quad .$ This equation applies whether the semiconductor is doped or not.

Consider an n-type semiconductor.  

$$n \cong n_e = N_{D+}$$

$$N_{D+} = N_D \quad \text{(Donor exhaustion)}$$

$$n \cong N_D \quad .$$

$$p = \frac{n_i^2}{n} = \frac{n_i^2}{N_D} \quad .$$









- Diodes, transistors, lasers, and LEDs are made using semiconductors. Silicon is the workhorse of very large scale integrated (VLSI) circuits.
- Forward bias Connecting a p-n junction device so that the p-side is connected to positive. Enhanced diffusion occurs as the energy barrier is lowered, permitting a considerable amount of current can flow under forward bias.
- Reverse bias Connecting a junction device so that the *p*-side is connected to a negative terminal; very little current flows through a *p*-*n* junction under reverse bias.
- Avalanche breakdown The reverse-bias voltage that causes a large current flow in a lightly doped p-n junction.
- Transistor A semiconductor device that can be used to amplify electrical signals.





- Superconductivity Flow of current through a material that has no resistance to that flow.
- Applications of Superconductors Electronic circuits have also been built using superconductors and powerful superconducting electromagnets are used in magnetic resonance imaging (MRI). Also, very low electrical-loss components, known as filters, based on ceramic superconductors have been developed for wireless communications.









